NR421 – Natural Resource Sampling

Double Sampling Answer Key

1.The data

Stratum	W_i	p_i	σ_i^2	σ_i
1	0.5	0.2	0.16	0.4
2	0.5	0.8	0.16	0.4
Population		0.5	0.25	0.5

a) The optimal sampling plan

$$v_i = \sigma_i\left[\left(\frac{c'}{c_i}\right)\frac{1}{\left(\sigma^2 - \sum W_i\sigma_i^2\right)}\right]^{1/2}$$

$$= 0.4\left[\left(\frac{0.25}{10}\right)\frac{1}{(0.25-0.16)}\right]^{1/2}$$

$$= 0.2108$$

$$n' = \frac{C^*}{c' + \sum c_i v_i w_i}$$

$$= \frac{3000}{0.25 + 2.108} = 127216 = 1273$$

$$n_i = n' v_i W_i = 1273(0.2108)(0.5) = 134.17 = 135$$

The double sampling plan is to sample 1273 units in the first phase and place the sample observation into one of two stratum. In the second phase, sub-sample 135 units in each stratum, The estimated sample variance under this plan is

$$V(\hat{p}_{st}) = \frac{1}{C^*}\left[\sum W_i\sigma_i\sqrt{c_i} + \left(\sigma^2 - \sum W_i\sigma_i^2\right)^{1/2}\sqrt{c'}\right]^2$$

$$= \frac{1}{3000}\left[1.2649 + (0.25-0.16)^{1/2}\sqrt{0.25}\right]$$

$$= 0.00066731$$

Compare to a simple random sample

$$n = \frac{C^*}{c} = \frac{3000}{10} = 300 \text{ samples at \$10 per sample}$$

$$V(\hat{p}) = \frac{pq}{n-1} = \frac{0.25}{299} = 0.0008361$$

b) The cost ratio for which double sampling is more precise

$$V(\hat{p}) > \frac{1}{C^*}\left[\sum W_i\sigma_i + \left(\sigma^2 - \sum W_i\sigma_i^2\right)^{1/2}\sqrt{c'}\right]^2$$

$$0.0008361 > \frac{1}{3000}\left[0.4 + \sqrt{0.09}\sqrt{c'}\right]^2$$

$$\left[\frac{(0.0008361(3000))^{1.2} - 0.4}{0.3}\right]^2 \geq c'$$

$$\frac{c'}{c} \leq 15.5$$

Double sampling will always be more precide.

2. a) average and total volume

$$\bar{y} = \left(\frac{3000}{6000}\right)8000 + \left(\frac{1000}{6000}\right)1000 + \left(\frac{2000}{6000}\right)100 = 4200$$

$$\hat{\tau} = 4200 * 1{,}000{,}000 = 4{,}200{,}000{,}000$$

b) Number acres sawtimber, pulpwood and non-commercial

$$\hat{p}_{saw} = \left(\frac{3000}{6000}\right)\left(\frac{280}{300}\right) + \left(\frac{1000}{6000}\right)\left(\frac{10}{50}\right) + \left(\frac{2000}{6000}\right)\left(\frac{2}{20}\right) = 0.533 * 1{,}000{,}000 = 533{,}333 \text{ acres}$$

$$\hat{p}_{saw} = \left(\frac{3000}{6000}\right)\left(\frac{20}{300}\right) + \left(\frac{1000}{6000}\right)\left(\frac{40}{50}\right) + \left(\frac{2000}{6000}\right)\left(\frac{8}{20}\right) = 0.300 * 1{,}000{,}000 = 300{,}000 \text{ acres}$$

$$\hat{p}_{saw} = \left(\frac{3000}{6000}\right)\left(\frac{0}{300}\right) + \left(\frac{1000}{6000}\right)\left(\frac{0}{50}\right) + \left(\frac{2000}{6000}\right)\left(\frac{10}{20}\right) = 0.166 * 1{,}000{,}000 = 166{,}667 \text{ acres}$$

c) volume of sawtimber and pulpwood

$$\bar{y}_{saw} = \left(\frac{3000}{6000}\right)\left(\frac{280}{300}\right)\left(\frac{2200000}{280}\right) + \left(\frac{1000}{6000}\right)\left(\frac{10}{50}\right)\left(\frac{5000}{10}\right) + \left(\frac{2000}{6000}\right)\left(\frac{2}{20}\right)\left(\frac{1500}{2}\right) = 3708.3$$

$$\hat{p}_{saw} = \left(\frac{3000}{6000}\right)\left(\frac{20}{300}\right)\left(\frac{200000}{20}\right) + \left(\frac{1000}{6000}\right)\left(\frac{40}{50}\right)\left(\frac{45000}{40}\right) + \left(\frac{2000}{6000}\right)\left(\frac{8}{20}\right)\left(\frac{500}{8}\right) = 491.7$$