Note on Aerodynamic Roughness-Parameter Estimation on the Basis of Roughness-Element Description

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1. Introductory remarks

It is not difficult to estimate fairly accurately, without detailed numerical analysis, the aerodynamic roughness parameter z_0 (cm) at a new micrometeorological site, after an anemometer mast has been installed and the first wind-profile data plot on semi-logarithmic graphs can be inspected. More challenging is the problem to estimate a z_0 value strictly based on a visual site survey and exclusively using a yardstick and metric measurements to describe the characteristic roughness elements. Recently I found myself challenged in this respect after having responded to an invitation for a brief visit at Davis, Calif., during one day of a two-week period in the spring of 1967, when the U. S. Army sponsored micrometeorological cooperative field experiment was in progress. The real challenge was that the experimental field, surrounding the unique subterranean Davis lysimeter, was dotted at that time by an unusual number of instrument stands, masts and towers brought to the site by the various participating groups. It occurred to me to ask if it would be feasible to separate a z_0 value due to the original grass cover from that possibly due to the various pieces of equipment at the site. It appeared that the effective overall aerodynamic roughness parameter of the field might have been larger during the days of the experiment than before and after. To answer this question some results of systematic micrometeorological research conducted during recent years at the University of Wisconsin, in the form of controlled roughness modification experiments on the ice of Lake Mendota, proved useful, as shall be shown below.

The determination of the roughness parameter with the aid of measured wind-profile data is also not without problems even when all instrumental errors are properly eliminated by anemometer matching and calibration. The problem arises from the fact that the true reference level of a logarithmic law (or its supplemented form for diabatic surface layers), is not *a priori* known, so that the determination of a "zero-plane displacement" is required in addition to z_0. Occasionally, there appears some confusion in the literature concerning these two lengths. To clarify the difference let it again be stated that z_0 is, ideally, at a given day, an absolutely fixed surface characteristic of the site, whereas the zero-plane displacement is the corrective height increment between the mathematically defined zero level of a numerical model of the wind profile and the arbitrary datum level from which the observer has measured anemometer heights (usually the physical ground surface at the convenient point just beneath the vertical array of anemometers, without due consideration of ground structure upwind of this point). An objective method for the simultaneous derivation of the two length parameters was developed and applied by Lettau (1957), later adapted by Robinson (1962) for machine computation, and more recently extended by Stearns (unpublished) to include the consideration of diabatic effects. The Lake Mendota experiments in Wisconsin, however, served readily to clarify the fundamental difference between the two length parameters experimentally. While z_0 could be artifically increased by obstacle layouts, the observer could control the displacement length independently by raising the entire mast array of anemometers vertically by a predetermined height increment. It follows that z_0 must be positive and invariant with respect to vertical mast positions, while the zero-plane displacement must vary by exactly the same arbitrary increment of height as the mast assembly was moved, either up or down.

2. Roughness-parameter formulas

For a recent summary of pertinent University of Wisconsin work, reference should be made to Lettau (1967a), dealing with general problems of micrometeorological control in out-of-doors experiments, including a discussion of Kutzbach's (1961) pilot experiment on wind-profile modification by fields of bushel baskets (several hundreds of them) laid out, and withdrawn systematically, on the ice of Lake Mendota. With such deliberate roughness modification upwind of an anemometer mast, the roughness parameter along about 50 m wind fetch was made to vary in a controlled fashion between about 0.01 cm and nearly 10 cm. The results of these experiments were combined with conventional wind tunnel data, involving for example, the aerodynamic drag properties of air foils due to rivet heads, etc. [see Schlichting (1965), or corresponding fluid dynamics texts]. In a slight simplification of Kutzbach's results, I proposed the following version of the relationship:

$$z_0 = 0.5h^* s/S, \qquad (1)$$

TABLE 1. Oversimplified estimates of roughness length z_0, for a systematic variation of h^* in Eq. (2) (corresponding approximately to heights of listed types of obstacles), and the resulting ratio h^*/z_0, if $z_0=0.058\ h^{*1.19}$.

h^* (cm)	Obstacle type	z_0 (cm)	h^*/z_0
1000	Forest trees, houses	214	5
100	Field crops, tall grasses	13.8	7
10	Lower grasses, weeds	0.80	11
1	Bare soils	0.058	17
0.1	Sand flats	0.0036	27

where h^* is the average vertical extent, or effective obstacle height (cm), s the silhouette area (cm^2) of the average obstacle, or area (measured in the vertical-crosswind-lateral plane) "seen" by the wind in the approach toward one characteristic individual obstacle, and S the specific area, or lot area (cm^2), measured in the horizontal plane or on the plane described by the local average earth/air interface. If n is the total number of roughness elements (or obstacles) on a site of total area A (cm^2), it follows that $S=A/n$. It can be suggested that the numerical factor of 0.5 in (1) corresponds to the average drag coefficient of the characteristic individual obstacle of silhouette area s. For a relatively large variety of S values, with an additional relatively small and supplementary control of h^* and s, Kutzbach's "bushel basket" experiments demonstrated that (1) produces z_0 estimates which agree normally within about ±25% with the results of detailed analysis of measured wind profiles.

Eq. (1) differs significantly from formulas employed previously in the meteorological literature; reference may be made to the summary in Sellers (1965, p. 150) showing that various authors used relationships of the form

$$\log z_0 = \log a + b \log h^*, \text{ i.e., } z_0 = a h^{*b}, \qquad (2)$$

where a and b are empirical constants (of which only b is dimensionless). Sellers (1965, p. 151) states that Tanner, Pelton, Kung and others have suggested b values between 0.99 and 1.42. If h^* is plant height in a field, the relationship (2) is claimed to express z_0 quite adequately.

Obviously, the appearance of only one metric parameter (h^*) is restrictive and it must be expected that (2) produces oversimplified results. To exemplify the practical verification of the two different z_0 formulas, let us first consider (2); Table 1 summarizes z_0 predictions, including the ratio h^*/z_0, if a regression equation derived by E. Kung [as referenced by Sellers (1965, p. 151)] is used.

Obviously, (2) fails to predict any differences in z_0 if one compares two sites on which roughness elements of the same h^* value occur in significantly different spatial distribution. For example, consider a uniform class of sizable roughness elements and two reference areas, of 600 m^2 each, on the same ice-covered lake, and assume that one is dotted by 500 roughness elements (which means relatively close spacing, or a lot area S of 1.2 m^2), while there are only 50 roughness elements of the same type on the other (which means a more generous lot area S of 12 m^2). According to (1), the area where $S=1.2$ m^2 will have a z_0 value 10 times as large as the area where $S=12$ m^2. However, (2) would predict the same z_0 value as long as h^* remains the same, unless there is independent justification for a change in the constants a or b. In another example, let us think of two, three or more distinct types or classes of roughness elements which coexist randomly on one site, each class having its characteristic yet different h^* value, but also different shape as well as lot area. It shall be shown that one can readily find specialized arrangements where the distribution is just so that each produces the same z_0 value when considered separately. Here again (2) would produce different and probably wrong results. Table 2 summarizes two examples of the first type, and one example of the second type.

The first case described in Table 2 corresponds closely to some of Kutzbach's Lake Mendota experiments mentioned before. The z_0 value of the undisturbed ice surface was slightly less than 0.015 cm. Hence (1) in combination with the data shown in Table 2 would suggest, for less than ~5 bushel baskets per 600 m^2, or

TABLE 2. Estimates of roughness length z_0 employing Eq. (1) for two cases of equal obstacle type but with two or three lot areas varied by a factor of 10; and one case of coexisting three different obstacle species as well as lot areas, but distributed such that the aerodynamic roughness length is the same.

Obstacle type, class	h^* (cm)	s (m^2)	S (m^2)	z_0 (cm)	h^*/z_0
Bushel baskets on lake ice	30	0.12	120	0.015	2000
			12	0.15	200
			1.2	1.5	20
Houses of a city, in dense vs loose array	500	50	100	125	4
			1000	12.5	40
Sand grains	0.1	1×10^{-6}	2×10^{-6}	0.025	4
Rocks and pebbles	10	0.01	2.0	0.025	40
Barchan dunes	250	20	1×10^{5}	0.025	10^{4}

a lot area in excess of 120 m^2, that the microstructure of the ice would be the dominating roughness. This is a very natural limit condition which demonstrates that beyond a certain lower threshold value of area density we cannot speak any more of the statistical action of roughness elements but must consider the individual air flow around each obstacle separately. The condition is comparable with other statistical statements, such as that 1 tree mi^{-2} is not a forest, but with $\gtrsim 1000$ trees mi^{-2} we certainly can speak of a forest.

The second case in Table 2 is a self-suggestive extrapolation of the bushel basket experiment. Geometrically similar but comparatively large roughness elements as houses in central residential sections are contrasted with ampler suburban spread due to larger lot areas, thus explaining the central city as the "rougher" neighborhood. The third case, ranging from grains to dunes, illustrates an application of (1) to conditions that correspond rather realistically to those which Prof. Stearns and myself encountered in the Peruvian desert (the "Pampa de la Joya") during a University of Wisconsin expedition devoted to micro- and mesometeorological studies of wind erosion; reference is made to Lettau (1967b, p. 12). It can be added that the wind profile structure in this desert suggested an "observed" z_0 value of 0.02 cm; see Lettau (1967a, p. 12). The highly intriguing result of equal z_0 contribution by obstacles ranging from grains to massive dunes (that is, a range of h^* over four powers of ten) may possibly be related to the fact that this desert is certainly windformed as evidenced by the perfect crescent shape of the barchans, which range in crest height from about 0.5 to 5 m; reference can be made to Hastenrath (1967). The silhouette area of these dunes was described by Lettau and Lettau (1969).

An independent testing of the "bushel-basket formula" (1) was recently reported by Weller (1968, p. 68) concerning the aerodynamic roughness parameter at a micrometeorological site on an antarctic inland-ice plateau near Mawson. According to Weller, the typical cusped blue ice surface was characterized by obstacle heights, silhouette areas, and lot areas as

$$h^* = 2 \text{ cm}, \quad s = 5 \text{ cm}^2, \quad S = 25 \text{ cm}^2,$$

whereupon (1) predicts $z_0 = 0.2$ cm. Weller evaluated wind profile data at this station and found a "measured" value of $z_0 = 0.23$ cm, which seems to be in fair agreement with the prediction.

With the above applications, the validity of (1) has been demonstrated for total area sizes A up to and including the mesoscale (10^9 m^2), with the lot-area S as well as the silhouette areas as important scaling factors. One limitation of (1) concerns the case where individual roughness elements of a given species are so densely packed that the ratio s/S approaches the order of unity; it even may become smaller than unity as for example in one of Kutzbach's pilot experiments in which the bushel baskets touched each other, thus forming a plateau-like and relatively smooth new or elevated "surface" at their tops. When approaching with the wind only the silhouette of the total array rather than that of individual obstacles could be seen. In fact, there are noteworthy similarities to problems of the visual range in air spaces partly obscured by foreign objects; examples are water droplets in a fog, or the tree-stems in the trunk space of a forest. In certain such cases the average visual range r can be estimated on the basis of simple geometry, in our notation as $r \approx h^* S/s$, if h^*/s is taken as the average width of the obstacle; in this case (1) can be reformulated as $z_0 \approx h^{*2}/2r$. For example, consider a cornfield where $h^* = 2$ m and the average visual range r is about 5 m; then z_0 could be estimated as about 0.4 m or 40 cm, which would appear to be the correct order of magnitude. For more realistic estimates, the vertical profile of horizontal and slant visual range, between the ground and the tips of the plants, must be known. The analogy serves to illustrate the limitation mentioned above, in that the statistical concepts of the visual range as well as roughness parameter would become invalid if the cross sections of the obscuring objects are considerably wider than that of the open air spaces in between.

Another limitation of (1) could possibly follow from restrictions concerning the horizontal size or scale of the total area A. However, it shall be shown that our z_0 formula can be useful for planetary boundary layer climatology even with rather extreme continental scales involved. Consider as a first example the highest of the States of the Union, Colorado, with an average altitude of 6800 ft, covering 1.0×10^5 mi^2, i.e., $A \cong 3 \times 10^{11}$ m^2. Since there are 1500 mountain peaks in Colorado which exceed 10,000 ft, $S = 2 \times 10^8$ m^2 is the average lot area of these huge "obstacles"; we let 10^3 m be their average height. Let us estimate their silhouette area by assuming that each obstacle has on the average a base length of 10^4 m which implies an average slope of the mountain sides of 1:5. Accordingly, the surface characteristics of this mountainous region (average height, silhouette area, and lot area) would be

$$h^* = 1000 \text{ m}, \quad s = 5 \times 10^6 \text{ m}^2, \quad S = 2 \times 10^8 \text{ m}^2,$$

whereupon (1) predicts $z_0 = 12.5$ m for such an extreme continental roughness. When V_g is the geostrophic speed and f the Coriolis parameter, the order of the surface Rossby number, $V_g/(z_0 f)$, for Colorado would be 10^4. According to tabulations in Lettau (1962) this magnitude is certainly low but still an acceptable border-line case for the employment of concepts of the geostrophic drag coefficient, and the subsequent calculation of frictional effects in the atmospheric boundary layer, including eddy dissipation of energy.

Let us finally remark briefly on Australia as an example of a truly continent-wide scale, where A is about 8×10^{12} m^2. For aerodynamic roughness estimates, in comparison with Colorado, it is evident that the average lot area of the relatively sparse mountain peaks in Australia is so small that their h^*s/S is insignificant in comparison with that for much lower but definitely more frequent roughness elements such as hills, dunes, and also perhaps certain vegetation forms. As was demonstrated above, (1) has the important advantage that it permits us to determine objectively that species among a variety of roughness types which will be the dominant contributor to the real z_0 value. In a more fundamental attack Lettau (1967b, pp. 5–8) has introduced spectral analysis of the total variance given by a topographical profile curve (for example, constructed by terrain-altitude readings, every 100 m or so, along the trajectory of the prevailing wind over distances of more than 100 km). Standard autocorrelation techniques produce a variance spectrum, as a function of topographical wavenumber, which readily serves to express objectively and numerically the relative importance of various topographical scales (for example, as given by knolls, ridges, mountains, to massifs) properly qualified by their geographic frequency.

3. Site roughness increase due to micrometeorological equipment

Let us return now to the specific small-area problem of the aerodynamic roughness length for the site of the micrometeorological cooperative field experiment at Davis, Calif. On the day of my visit (29 April 1967), I estimated for the fescue grass cover the following characteristics:

$$h^*=10\text{ cm},\quad s=4\text{ cm}^2,\quad S=16\text{ cm}^2,$$

whereupon (1) predicts $z_0=1.25$ cm. Incidentally, if mowing of a lawn would reduce h^*, let's say by 25%, the silhouette area would be reduced by about the same amount (or more, e.g., if "tassels" are trimmed; or less, if only relatively thin tips are clipped). The lot area, of course, remains unaltered so that according to (1) the total effect would be a reduction of z_0 by ~50%.

A visual impression was that perhaps the assembled instrument masts and stands contributed just as much to the effective aerodynamical roughness of the site as the difference between the unmowed and mown grass. On an area where A was ~4000 m^2, I counted a total number of forty ($n=40$) distinct "obstacles"; in other words, each individual instrument mast, stand, tower, etc., occupied on the average a lot area of about $A/n=S=100$ m^2. According to the results of bushel basket experiments discussed in connection with Table 2, and in view of the properties of the fescue grass, this S value could be critical, depending on the other two characteristics h^* and s. Since there was no micrometeorological measurement activity on that day, I walked from instrument stand to instrument stand and estimated, with the aid of a yardstick, each height h^* as well as silhouette area s. The latter estimate was problematic, since the metric evaluation should have been weighted by the aerodynamic quality of the instrument stand, or its form-drag coefficient. This was felt to be especially important for those profile masts which were reasonably slender but supported by an intricate system of guy wires. The results, summarized in Table 3, can be taken only as indicating orders of magnitude; a deliberate attempt was made to remain on the conservative side in the judgment of individual silhouette areas. According to Table 3 and the size of the area, the following data set characterizes the aerodynamic roughness properties of the site instrumentation during the cooperative field experiments at Davis:

TABLE 3. Results of estimates of characteristic height (h^*) and silhouette area (s) of the total of 40 instrument masts and stands on the area (~4000 m^2) of the micrometeorological field experiment at Davis, Calif., on 29 April 1967.

Range of h^* (cm)	Number	Class average of h^* (cm)	Class average of s (m^2)
<16	5	10	0.008
16– 50	8	40	0.070
51–200	10	120	0.140
201–400	11	300	0.350
401–800	5	600	0.600
>800	1	1500	1.500
Weighted mean		240	0.25

$$h^*=240\text{ cm},\quad s=0.25\text{ m}^2,\quad S=100\text{ m}^2,$$

whereupon (1) yields $z_0=0.3$ cm. This may represent an underestimate because guy wire resistances were not considered as weight factors in determining the effective silhouette area for wind drag. If (2) would be applied with the numerical coefficients as specified in the caption of Table 1, the result would be $z_0=41$ cm for $h^*=240$ cm which obviously is an entirely unrealistic value.

Before I departed from California on 30 April 1967, I wrote down my results and some brief comments including in part those presented in this section. It was appreciated that the field coordinator took good care of the manuscript and its immediate distribution in mimeographed form among the participants, under the title "Essay on the aerodynamic roughness of the site of the 'MiMeEx' at Davis, California, 1967." Comprehensive analyses of the large amount of profile data in connection with detailed measurements of boundary fluxes are still in progress. But it can be mentioned that Prof. C. Stearns (who at Davis was in charge of the University of Wisconsin group responsible for wind and temperature profile measurements, with the aid of three identical anemometer-thermometer masts forming a site-circumscribing triangle) informed me that "measured" z_0 values varied within the limits of 0.8–1.4 cm during the duration of the experiment. This *a posteriori* overall result appears to be in satisfactory agreement with the *a priori* estimate made without the benefit of profile analysis. More detail will be reported by Prof. Stearns in a forthcoming paper.

4. Conclusions

Eq. (1) should only be used when the obstacle field is of sufficient extent in the upwind direction of an anemometer mast. This condition is, of course, not satisfied in the particular application of Section 3 where the anemometer mast itself is assumed to act as a roughness element, while other masts will be in an upwind position only for certain wind directions. At distances beyond a few hundred meters, the Davis site is surrounded by substantial obstacles to air flow, like lines of trees; a consideration of their effects should be incorporated in a more detailed study. The question remains whether the z_0 value due to the supplementary equipment on the site should be algebraically added to the original z_0 of the natural grass cover, or should the composite z_0 value be calculated as a weighted mean between the two individual contributions. This question could perhaps be clarified by a new set of controlled micrometeorological out-of-doors experiments employing two distinct classes of roughness elements, at varying absolute and relative area densities, on the same field. Recalling in the U. S. system of dry measures that 1 bushel (equivalent to 35.2 liters) equals 64 pints, the proposed new experiments could consist of intermingling "bushel baskets" with "pint baskets." It may be concluded that a combined "bushel and pint basket formula" for aerodynamical roughness may be useful for expressing the quality of a site as a micrometeorological test area. Furthermore, roughness parameter estimates could be made more securely than before, for any continental area for which topography and land use are defined. A more fundamental attack to this problem will require that consideration of obstacle geometry be supplemented in two respects: (i) the use of variance spectra of topographical profiles as pioneered by Lettau (1967b), and (ii) the determination of truly representative coefficients of form-drag for the individual obstacle type, rather than an "average" coefficient of $\frac{1}{2}$. Knowledge of the z_0 value, in combination with a given overall horizontal pressure gradient, would yield the surface-Rossby number and, subsequently, as shown by Lettau (1962) via the geostrophic drag coefficient, an estimate of the surface shearing stress as well as energy dissipation in the atmospheric boundary layer.

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